A Rapid Response Impact Tube in Two-Phase Flow

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In a study of mercury-nitrogen cocurrent bubbly flow, Neal and Bankoff (*1*) measured time-averaged pressures on an impact tube. They found that for low nitrogen volume fraction the time-averaged dynamic pressure was nearly double the dynamic pressure for an equal mass velocity mercury flow without nitrogen bubbles. They argued that the higher pressure resulted from the liquid's acting as a series of liquid slugs separated by bubbles which, since they are not appreciably slowed down before impinging on the probe, generate an impulse pressure equal to the liquid density times its velocity squared.

EXPERIMENT

Since this result is rather surprising, an experiment was planned in which both time-averaged and nearly instantaneous probe pressure could be measured. A 1/32 I.D. stainless capillary impact (or total pressure) tube was inserted into an air-water mixture flowing upward in a plexiglas tube. For time-averaged dynamic pressure reading this probe was attached to one leg of an inclined manometer while the other leg was connected to a static pressure hole at the tube wall. Rapid variation in total pressure was sensed by attaching the probe directly to a quartz piezoelectric pressure transducer. For both arrangements bubbles were prevented from entering the probe by a slow purging stream of water metered through a coil of 0.015 in. I.D. capillary tubing. The flow over the probe and the oscilloscope trace indicating the measured total pressure were simultaneously photographed using a standard Fastex 16 mm. camera.

RESULTS

It was expected that before the initial contact of a bubble the probe would indicate the total head of the liquid (static plus dynamic pressure). Further, when a bubble passed over the probe tip, the total head would decrease by the liquid dynamic head since the air density is negligible relative to the liquid density. Figure 1 shows a typical oscilloscope trace reproduced from the Fastex record for a flow with small air volume fraction. In this flow, bubble impinging and departure on the probe were visible; they have been marked on the figure. Several points are of interest. The amplitude of the pressure drop was within 10% of the local liquid dynamic pressure as calculated from the observed bubble velocity (assuming no local slip). The rapidity of the pressure drop was consistent with an approximate probe response time calculation. The lag in pressure return is unexplained. The oscillation in pressure reading between bubbles is a static pressure oscillation due primarily to bubble collisions with the nearby probe support.

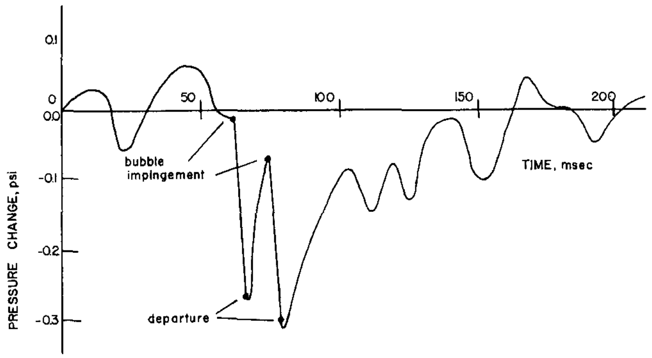

Fig. 1. Rapid response probe signal.

Based on these observations one would expect the time averaged dynamic pressure in two phase flow to be less than $\frac{1}{2}\rho V^2$ where V is the true average liquid velocity, and this was verified. The probe was connected to the inclined tube manometer and readings taken at different radial and axial positions for air-water flow with from 25 to 60% air volume fraction and superficial liquid velocity of 6 ft./sec. The time-averaged dynamic head read on the manometer was converted to an indicated liquid velocity by the usual dynamic head formula ($\frac{1}{2}\rho V^2$). For known air, Q_V, and water, Q_L, volume flow rates Marchaterre and Hoglund (*2*) give expected slip ratio, S, defined as the ratio of air to water velocity. The average liquid velocity may then be calculated from $\overline{U} = (Q_V + SQ_L)/SA_t$. The ratio of the liquid velocity indicated by the impact probe to the local calculated liquid velocity ranged from 0.45 to 0.79 with higher values occurring farthest downstream from the mixing location.

The agreement which is seen to exist between the instantaneous and time-averaged dynamic pressures shows that for air-water flows the theory of Neal and Bankoff does not hold.

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Gas Holdup of a Bubble Swarm in Two Phase Vertical Flow

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The rise of a gas bubble in a confined liquid medium is somewhat analogous to the corresponding sedimentation of a solid particle; but this analogy has been misinterpreted in the past by various authors who were therefore obliged either to set narrow operating limits on the validity of their equations (*3*) or to correct them by means of empirical factors (*4*). Marrucci (*7*) derived an expression for the rise velocity of a swarm of bubbles and concluded that the rise velocity showed a weaker dependence on the fraction of solid in suspension than the settling velocity of a multiparticle suspension of rigid spheres following Happel's free surface model (*8*) (Figure 1.). He noted qualitatively that the sparse data of Nicklin (*10*) and his own data indicated a relatively small decrease in rise velocity with increasing ϵ, but quantitatively his model failed to agree with the experimental data of various other investigators. Although no explicit mention is made of the radius of bubbles which make up the swarm, Marrucci's model does implicitly indicate the effect of bubble radius on the rise velocity of the swarm.

Mendelson (*6*) recently derived an equation for predicting the rise velocity of single bubbles in infinite media from the interfacial disturbance analogy between the motion of waves on an ideal liquid and the rise of bubbles in a low viscosity liquid.

$$V_\infty = \sqrt{\frac{\sigma}{\rho r_e} + g r_e} \qquad (1)$$

This equation was found to represent the experimental data well for the rise velocity of single bubbles larger than 1.5 mm. in diameter.

Later, Maneri and Mendelson (*5*) extended this analogy for predicting the rise velocity of large single bubbles in bounded media. They showed that for large N_{Eo}, that is for large tube diameters such that $1/N_{Eo} << 1$, the rise velocity of a single bubble could be represented by

$$V/V_\infty = \sqrt{\tanh [0.25(r/r_e)]} \qquad (2)$$

Equation (2) was found to correlate experimental data well for $1/\lambda = (r/r_e)$ between 1 and 10, the lower value of which corresponds to a slug.

In this paper an attempt is made to extend Mendelson's wave analogy further, to describe the rise velocity of a swarm of bubbles, by employing the cell model technique proposed by Happel and Ast (*9*) for sedimentation and subsequently used by Marrucci (*7*). Following their approach we have the necessary relationship between λ and gas volume fraction, ϵ, given by

$$\epsilon = \lambda^3 \qquad (3)$$

Now combining Equations (2) and (3) we get

$$V_o = V_\infty \sqrt{\tanh [0.25(1/\epsilon)^{1/3}]} \qquad (4)$$

Thus Equation (4) would predict the energy destroying velocity [as defined by Nicklin (*10*)] of a bubble swarm in tubes of large diameters. It should, however, be pointed out here that Equation (4) will not give the true rise velocity, V_o, for slugs and is applicable only for a swarm consisting of large spherical bubbles. But Equation (2), with $r = r_e$, does represent the rise velocity of slugs in a quiescent liquid medium, for which it then simplifies (*5*) in combination with Equation (1) for large N_{Eo} to

$$V_s = 0.35\sqrt{gD} \qquad (5)$$

which is the familiar Dumitrescu equation (*11*). Equation (5) should, however, be modified as suggested by Nicklin (*10*), to be used for predicting the energy destroying velocity, V_o, for slugs:

$$V_o = 0.2\ (u_g + u_L) + V_s \qquad (6)$$

The gas velocity, u_g, for which a certain gas volume fraction, ϵ, would exist at the given liquid flow rate, u_L, is given by

$$u_g = \frac{\epsilon(V_o + u_L)}{(1 - \epsilon)} \qquad (7)$$

a relationship derivable directly from Nicklin's equations. The rise velocity of the bubble swarm will then be given by

$$V_B = V_o + u_g + u_L \qquad (8)$$

DISCUSSION

Equation (4) represents the simplest expression that can be derived for the rise velocity of bubble swarms. Equation (4) predicts the ratio V_o/V_∞ to be a function of gas holdup, ϵ, only, but V_o itself depends on bubble diameter as well as on gas holdup. Figure 1 shows a plot of Equation (4) along with plots of Marrucci's equation for bubble swarms and Happel's equation for sedimentation of solid spheres. The discrepancy between the curves for bubble swarms and solid particles arises from the fact that the tangential liquid velocity is zero at the surface of a solid particle but not zero at the surface of a bubble. As a result, the energy dissipation is smaller and therefore the relative velocity higher for the bubble swarm. The reason

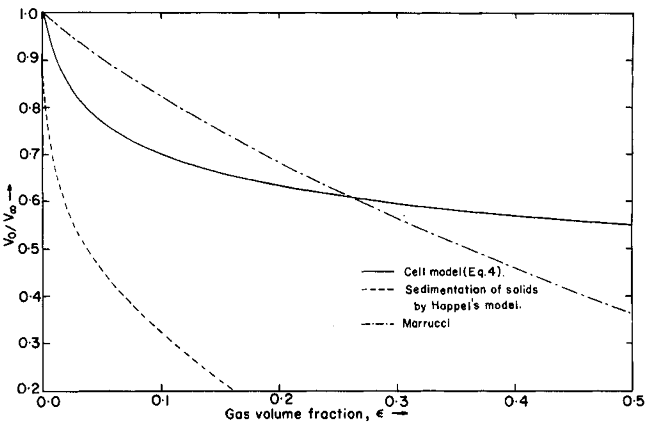

Fig. 1. Effect of gas volume fraction on the rise velocity of bubble swarms.

for the discrepancy between the two models for rise velocity of a bubble swarm is that Marrucci based his derivation on potential flow theory, whereas the model described herein is based on Mendelson's wave analogy (*5*). Nicklin reported data for V_o as a function of u_g, but did not report the bubble diameter as a function of gas velocity. No reliable correlation exists for predicting the diameter of bubbles that form the swarm. But the operating conditions of Nicklin probably gave rise to bubble diameters between 2 and 6 mm. (*14*).

To check the validity of the above equations, we need systematic data for gas holdup, average bubble diameter, and a description of flow regimes as a function of gas and liquid velocities. Figure 2 shows data of Bridge, et. al. (*12*), Nicklin (*10*), and Deraux (13) for the air-water system with zero net water flow, compared with the present model [Equation (1), (4), and (7)] for a bubble diameter of 3 mm. Although average diameters of bubbles are not reported (except for data of Bridge) we see that the theoretical relationship for 3 mm. diameter bubbles does follow the data well, whereas the empirical correlation of Davidson, et. al. (*15*) follows the data only at the lower gas velocities corresponding to the bubble flow regime.

The experimental data of Ellis and Jones (*19*) and Towell and Strand (*20*) are of special interest since they employed large column diameters and therefore introduced the additional phenomenon of recirculation, which is absent in smaller pipes (less than 4 in. in diameter). Towell and Strand (*19*) used a 16 in. diameter column and reported that bubble diameter was essentially constant (0.21 in.) for all gas rates except the highest (1.0 ft./sec.), where occasional large irregular masses of gas were observed. Ellis and Jones (*20*) used circular pipes of 1, 2, 4, and 12 in. diameters and reported that, although no time slugs (gas bubble bridging the pipe diameter) were observed in pipes greater than 4 in. in diameter, large bubbles and irregular masses of gas appeared to be moving up these columns. In pipes up to 2 in. in diameter, slugs were reported to appear at gas velocities exceeding 0.1 ft./sec. Ellis and Jones gave an empirical correlation for the transition from bubble to slug flow as

$$u_g = 0.2\, u_L + 0.1$$

Therefore it seems reasonable to conclude that bubbles of up to about 2 in. in diameter could be present in vertical two-phase flow depending on gas flow rate and pipe diameter.

Since the model described herein is dependent on the bubble diameter, it is necessary to know the time average diameter of the bubble in the swarm in order to correctly predict the gas holdup. Also it is important to note that since the model does not account for bulk recirculation, which is present in large diameter columns (*19, 20*), it

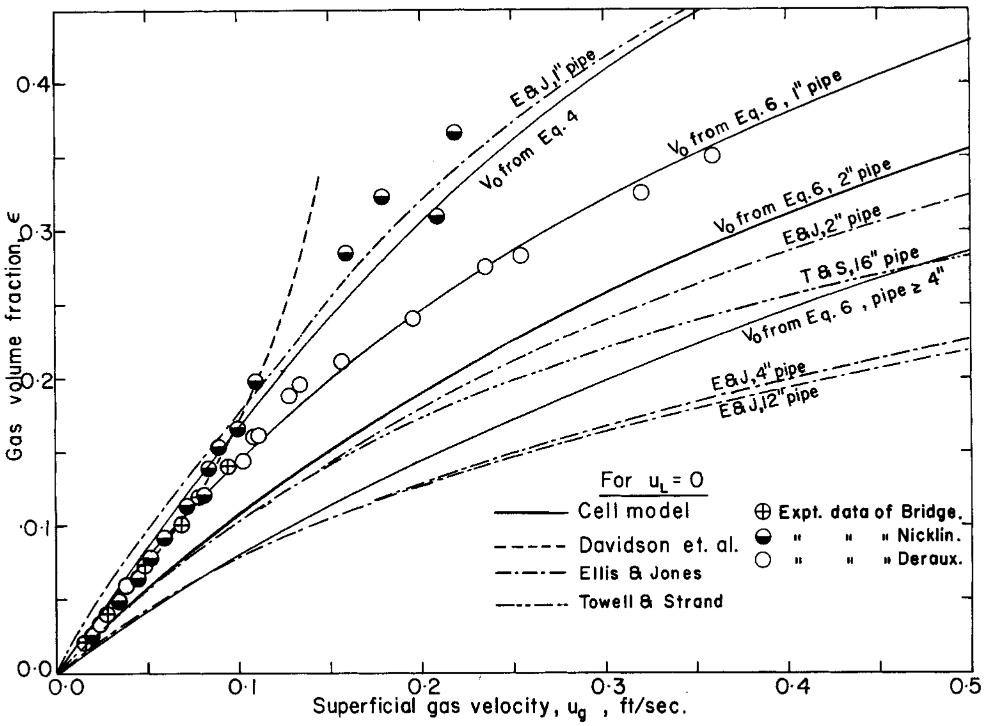

Fig. 2. Gas holdup for noncirculating air-water system.

will therefore predict higher gas holdup than actually observed in columns where recirculation is prevalent. This is illustrated by the large column data of Ellis and Jones and of Towell and Strand, which are plotted in Figure 2 and fall below that predicted by the cell model. It may be mentioned here that since the bubble diameter in the swarm is increasing gradually with the gas rate, it is not possible to satisfy the experimental data with a single bubble size in the cell model equation for all gas rates. Therefore to predict the gas holdup in large diameter columns, one must know both the bubble size and the liquid recirculation velocity as a function of gas rate.

We can also use Equation (4) to predict gas holdup for cocurrent gas-liquid flow. Baker and Chao (*17*) reported that the relative velocity for bubble rise in a turbulently flowing liquid is the same as for a quiescent liquid. Thus V_o for cocurrent gas-liquid flow could be obtained from Equations (1) and (4). For cocurrent gas-liquid flow Patrick (*18*) reported the average bubble diameter, d_e, to be a function of true liquid velocity V_T.

$$d_e = 0.52/V_T^{0.666} \quad (0.5 < V_T < 5.0) \tag{9}$$

where

$$V_T = u_L/(1-\epsilon) \tag{10}$$

Dukler, et. al. (*16*) compiled all the existing data on two-phase flow and checked the validity of various proposed empirical correlations. They found that Hughmark's (*2*) correlation represents the existing data better than any other correlation. Therefore it was decided to check the above theory with Hughmark's correlation. Figure 3 shows this comparison, as well as Ostergaard's Equation (1) and Deraux's data for a superficial liquid velocity of 0.569 ft./sec. The present method, using the correlation for bubble diameter reported by Patrick, agrees reasonably well with Hughmark's correlation for higher liquid velocities. We should use Equation (4) for the bubble flow regime and Equation (6) for the slug flow regime. Deraux (*13*) reported that slugs begin to appear at a gas volume fraction of about 0.2, whereas Patrick (*18*) observed that no slug formation occurs until $\epsilon = 0.4$. Using the proper equation for each regime we see that the deviation between the present correlation and Hughmark's correlation is reduced appreciably.

CONCLUSIONS

A method has been derived to predict the gas holdup of a swarm of bubbles based on the cell technique for representing a swarm, using the bubble velocity in bounded media as developed by Mendelson (*5*). The method is applicable to low viscosity and comparatively pure gas-liquid systems. The presence of any interfacial impurity will tend to make the gas bubbles behave like solids and the equation will not be applicable. The model shows good agreement with existing experimental data

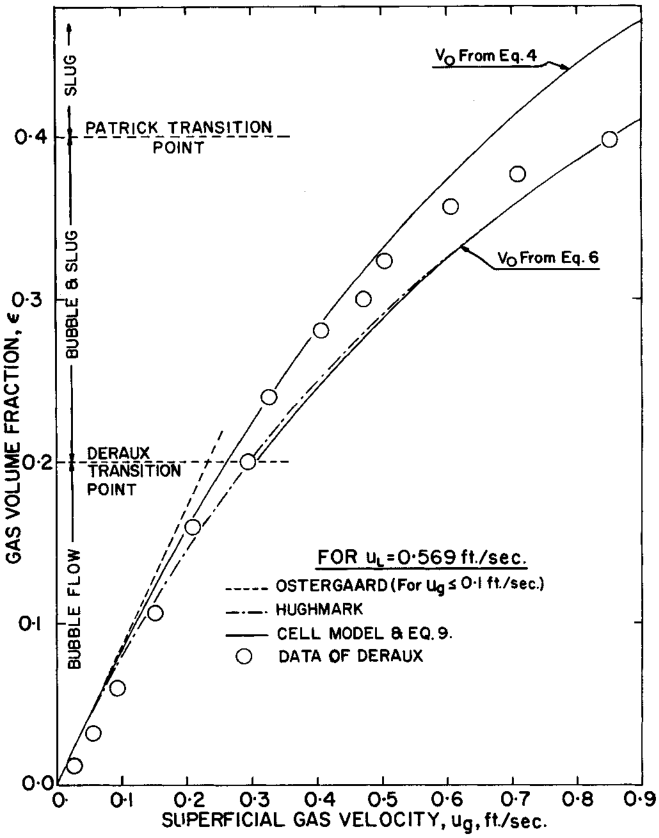

Fig. 3. Gas holdup for cocurrently flowing air-water system.

for the air-water system and gives better insight into rising bubble swarms than models based on the hindered settling of rigid particles. To check the general validity of the above model, systematic data for average bubble diameter, gas holdup, and flow regime as a function of gas and liquid velocities are needed.

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NOTATION

D = diameter of tube, ft.
d_e = average bubble diameter, cm.
g = acceleration of gravity, ft./sec.^{2}
N_{Eo} = Eötvös number based on tube radius, $g\rho r^2/\sigma$
r = tube radius, ft.
r_e = equivalent bubble radius based on a sphere of equal volume, ft.
u_g = superficial gas velocity based on empty tube area, ft./sec.
u_L = superficial liquid velocity based on empty tube area, ft./sec.
V = rise velocity of single bubble in a tube, ft./sec.
V_o = energy destroying velocity of bubble swarm, ft./sec.
V_B = velocity of bubbles relative to wall for steady co-current gas-liquid flow, ft./sec.
V_∞ = rise velocity of single bubble in infinite media ft./sec.
V_s = slug rise velocity, ft./sec.
V_T = true liquid velocity as defined by Equation (10), ft./sec.

Greek Letters

ρ = density of liquid phase, lb./cu.ft.
σ = surface tension, poundals/ft.
λ = ratio of radii, r_e/r
ϵ = gas volume fractions
$\Delta\rho$ = difference between liquid and gas density, lb./cu.ft.

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Latent Heat Estimation Using Altenburg's Quadratic Mean Radius

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An array of theoretical and semi-empirical relations is available (*1* to *3*) to predict latent heats of vaporization for pure substances. These methods as a rule use parameters associated with the vapor pressure data, critical data and molecular data (*4*), such as molecular weight and molecular volume. The difficulties in using the vapor pressure and critical point data are well known (*1*, *3*). On the other hand, methods based on the molecular weight and molecular volume (*4*) are not entirely satisfactory in the case of complex isomeric substances. It appears that a parameter describing molecular mass distribution as related to the configuration of the molecule, would provide a direct and useful way to correlate the latent heat of vaporization not only for the simple but also for these structurally branched (isomeric) compound series (for instance, alkane isomers).

Such a parameter is Altenburg's (*5* to *8*) quadratic mean radius $\overline{R_v^2}$ $\left(\overline{R_v^2} = \frac{1}{N^2}\sum_{i=1} n_i a_i^2\right.$, where n_i is the frequency of the distance a_i^2 occurring in the molecule and N is the number of the same kind atoms in the molecule$\left.\right)$. This parameter is associated with the mean moment of inertia of molecule (*9*) and so, connecting the molecular mass distribution with the particular geometry of the molecule permits to distinguish between the various isomers within the given compound class.

Using the published values of $\overline{R_v^2}$ (Table 1) and the available experimental values for the latent heat of vaporization as taken from Rossini (*10*),* the following correlating equation type was established:

$$\Delta H_{vb} = qX^b \qquad (1)$$

where $X = \overline{R_v^2}/a_i^2$ and q and b are constants referring to a particular class of compounds. The conventional least squares treatment was used to evaluate the constants q, b

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* Tables 3, 4, 5, and 6 presenting computed [from Equation (4), using constants found in Table 2] and experimental values of ΔH_v for n-alkanes, cyclic alkane, and benzene compounds and alkane isomers; are deposited as document NAPS-00318 with the ASIS National Auxiliary Publications Service, c/o CCM Information Sciences, Inc., 22 W. 34th St., New York 10001 and may be obtained for $1.00 for microfiche or $3.00 for photocopies.